Real Number

Quick Revision

Prime, Coprime and Composite numbers

Prime numbers are those numbers, which have no factors other than 1 and the number itself.

e.g. 2, 3, 5, 7, 11, ...

Coprime numbers are those numbers, which do not have any common factor other than 1.

e.g. 2 and 9 are coprime numbers.

Composite numbers are those numbers, which have atleast 1 factor other 1 and the number itself

e.g. 4, 6, 24, ...

(i) 1 is neither prime nor composite.

(ii) 2 is the smallest prime number.

Factor Tree

A chain of factors which is represented in the form of a tree, is called factor tree.

Fundamental Theorem of Arithmetic

Fundamental theorem of arithmetic states that every composite number can be written (factorised) as the product of primes and this factorisation is unique, apart from the order in which the prime factors occur. It is also called unique factorisation theorem.

$\therefore$ Composite number = Product of prime numbers

Relation between Numbers and their HCF and LCM

(i) For any two positive integers a and b, the relation between these numbers and their HCF and LCM is

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

$$\Rightarrow \quad \text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

$$\text{or} \quad \text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

(ii) For any three positive integers a, b and c, the relation between these numbers and their HCF and LCM is

$$\text{HCF}(a, b, c) = \frac{a \times b \times c \times \text{LCM}(a, b, c)}{\text{LCM}(a, b) \times \text{LCM}(b, c) \times \text{LCM}(c, a)}$$

$$\text{or} \quad \text{LCM}(a, b, c) = \frac{a \times b \times c \times \text{HCF}(a, b, c)}{\text{HCF}(a, b) \times \text{HCF}(b, c) \times \text{HCF}(c, a)}$$

Real Numbers

A number, which is either rational or irrational, is called a real number.

Rational Numbers

A number that can be expressed as $\frac{p}{q}$, where p, q are integers and $q \neq 0$, is called a rational number.

e.g. $\frac{3}{5}, \frac{7}{9}, \frac{13}{15}, \frac{-7}{3}$, etc.

Irrational Numbers

A number that cannot be expressed in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$, is called an irrational number.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{15}, \pi, -\frac{\sqrt{2}}{\sqrt{3}}$, 0.10110111011110111..., etc.

Useful Theorems

Theorem 1 Let p be a prime number and a be a positive integer. If p divides a^2, then p divides a.

Theorem 2 $\sqrt{2}$ is irrational number, then $2\sqrt{2}$ is irrational number.

Decimal Expansions of Rational Numbers

The decimal expansion of every rational number is either terminating or non-terminating repeating.

Terminating Decimal Expansion

The number which terminates (i.e. ends completely) after a finite number of steps in the process of division, is said to be terminating decimal expansion. e.g. 1.25, 3.14, etc.

Non-terminating Decimal Expansion

The number which does not terminate in the process of division, is said to be non-terminating decimal expansion.

There are following two types of non-terminating decimal expansions

(i) **Non-terminating Repeating Expansion** The number, which does not terminate but repeats the particular number again and again in the process of division, is said to be non-terminating repeating decimal or recurring decimal expansion. The repeated digit is denoted by bar '–'

e.g. $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$

(ii) **Non-terminating Non-repeating Decimal Expansion** The number, which neither terminates nor repeats the particular number in the process of division, is said to be a non-terminating non-repeating decimal expansion. These numbers are called irrational numbers.

e.g. $1.0300300003\ldots, \sqrt{3}$, etc.

Important Theorems on Decimal Expansion of Rational Numbers

Theorem 3 Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form p/q, where p, q are coprimes and the prime factorisation of q is of the form $2^n 5^m$, where n and m are non-negative integers.

Theorem 4 (Converse of Theorem 3) Let $x = p/q$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n and m are non-negative integers. Then, x has a decimal expansion, which terminates.

Theorem 5 Let $x = p/q$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n and m are non-negative integers. Then, x has a decimal expansion, which is non-terminating repeating (recurring).

Objective Questions

Multiple Choice Questions

1. For some integer m, every even integer is of the form

(a) m (b) $m+1$
(c) $2m$ (d) $2m+1$

2. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible, is

(a) 6 (b) 8
(c) 12 (d) 24

3. The product of two consecutive positive integers is divisible by 2'.

(a) True (b) False
(c) Can't say (d) Partially True/False

4. The number, 4^n, where n is a natural number, ends with the digit 0 for any natural number n.

(a) True
(b) False
(c) Can't say
(d) Partially True/False

5. 12^n ends with the digit 0 or 5 for natural number n

(a) 2 (b) 3
(c) No value (d) 5

6. $(3 \times 5 \times 7) + 7$ is a

(a) Prime number (b) Composite number
(c) Can't say (d) None of these

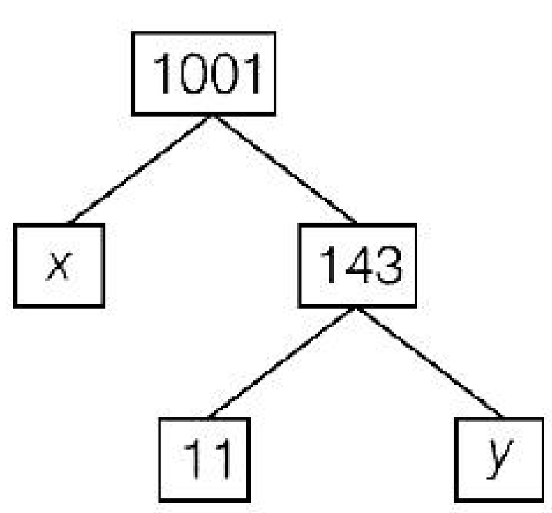
7. The number $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 5$ is ……… number.

(a) Prime (b) Composite
(c) Can't say (d) None of these

8. The total number of factors of a prime number is ***[CBSE 2020]***

(a) 1 (b) 2 (c) 0 (d) 3

9. The values of x and y is the given figure are



(a) 7, 13 (b) 13, 7
(c) 9, 12 (d) 12, 9

10. HCF of 96 and 404 is equal to ……… .

(a) 2 (b) 3 (c) 4 (d) 5

11. Total number of distinct primes in the prime factorization of number 27300.

(a) 5 (b) 7
(c) 13 (d) 21

12. The unit place digit of HCF of $2^2 \times 3^2 \times 5^3 \times 7, 2^3 \times 3^3 \times 5^2 \times 7^2$ and $3 \times 5 \times 7 \times 11$ is

(a) 70 (b) 105 (c) 175 (d) 225

13. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is ***[NCERT Exemplar]***

(a) xy (b) xy^2
(c) x^3y^3 (d) x^2y^2

14. Write the HCF of the smallest composite number and the smallest prime number.

(a) 1 (b) 2
(c) 3 (d) 4

15. Two numbers are in the ratio of 15 : 11. If their HCF is 13, then numbers will be

(a) 195 and 143
(b) 190 and 140
(c) 185 and 163
(d) 185 and 143

16. Product of two coprime numbers is 117 their LCM should be ……… .

(a) 1
(b) 117
(c) equal to their HCF
(d) lies between 100 – 110

17. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) ***[NCERT Exemplar]***

(a) 10
(b) 100
(c) 504
(d) 2520

18. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then find the value of m. ***[NCERT Exemplar]***

(a) 1 (b) 2
(c) 3 (d) 4

19. If LCM of 12 and 42 is $10m + 4$, the value of m is equal to

(a) 7 (b) 8
(c) 6 (d) 9

20. There are 24 peaches, 36 apricots and 60 bananas and they have to arranged in several rows in such a way that every rows contains the same member of fruits of only one type. What is the minimum number of rows required for this to happen?

(a) 12 (b) 9
(c) 10 (d) 14

21. Four bells toll at intervals of 10 s, 15 s, 20 s and 30 s respectively. If they toll together at 10 : 00 am at what time will they toll together for the first time after 10 am?

(a) 10 : 01 am
(b) 10 : 02 am
(c) 10 : 00 : 30 am
(d) 10 : 00 : 45 am

22. P is the LCM of 2, 4, 6, 8, 10; Q is the LCM of 1, 3, 5, 7, 9 and L is the LCM of P and Q. Then, which of the following is true?

(a) $L=21P$ (b) $L=4Q$
(c) $L=63P$ (d) $L=16P$

23. The LCM and HCF of 120 and 144 by fundamental theorem of arithmetic is

(a) 720, 22 (b) 720, 24
(c) 640, 24 (d) 640, 22

24. If HCF of two numbers is 2 and their product is 120, find their LCM.

(a) 120 (b) 60
(c) 240 (d) 80

25. If the HCF of 65 and 117 is 13, LCM of 65 and 117 is $45 \times a$, then the value of a is

(a) 9 (b) 11
(c) 13 (d) 17

26. If HCF $(a, b) = 12$ and $a \times b = 1800$, then LCM $(a, b) =$

(a) 3600 (b) 900
(c) 150 (d) 90

27. If HCF of 306 and 657 is 9, then their LCM is ....

(a) 22338 (b) 23328
(c) 22833 (d) 33228

28. The LCM and the HCF of two numbers are 1001 and 7 respectively. How many such pairs are possible?

(a) 0 (b) 1
(c) 2 (d) 7

29. If p is prime, then HCF and LCM of p and $p+1$ would be

(a) HCF $= p$, LCM $= p+1$
(b) HCF $= p(p+1)$, LCM $=1$
(c) HCF $=1$, LCM $= p(p+1)$
(d) None of the above

30. The HCF and LCM of 12, 21 and 15 respectively, are **[CBSE 2020]**

(a) 3, 140 (b) 12, 420
(c) 3, 420 (d) 420, 3

31. In which of the following is a irrational number?

(a) $\frac{22}{7}$ (b) 3.1416
(c) $3.\overline{1416}$ (d) 3.141441444...

32. The number of irrational numbers between 15 and 18 is infinite.

(a) True
(b) False
(c) Can't say
(d) Partially True/False

33. The product of a non-zero rational and an irrational number is

(a) always irrational
(b) always rational
(c) rational or irrational
(d) one

34. $3.\overline{27}$ is

(a) an integer number (b) a rational number
(c) an irrational number (d) None of these

35. A rational number p/q has a terminating decimal expansion of prime factorization of q have

(a) 3 (b) 2
(c) 5 (d) Both (b) and (c)

36. If $\frac{13}{125}$ is a rational number, then decimal expansion of it, which terminates

(a) 0.104 (b) 1.04
(c) 0.0104 (d) 0.140

37. On the basis of form $2^m \times 5^n$ of denominator that $\frac{1458}{1250}$ will be expanded in decimal upto places will be

(a) one (b) two (c) three (d) four

38. After how many places, the decimal form of $\frac{125}{2^4 \cdot 5^3}$ will terminate?

(a) Three places (b) Four places
(c) Two places (d) None of these

39. Without actually performing the long division, the terminating decimal expansion of $\frac{51}{1500}$ is in the form of $\frac{17}{2^n \times 5^m}$, then $(m + n)$ is equal to

(a) 2 (b) 3 (c) 5 (d) 8

40. The rational number which can be expressed as a terminating decimal number will be

(a) $\frac{77}{210}$ (b) $\frac{129}{2^2 \times 5^7 \times 7^5}$
(c) $\frac{13}{3125}$ (d) $\frac{8}{17}$

41. Which of the following rational numbers have non-terminating repeating decimal expansion?

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$
(c) $\frac{23}{200}$ (d) None of these

42. The rational number $\frac{124}{164}$ can be expressed as a .... decimal number.

(a) non-terminating (b) can't say
(c) terminating (d) None of these

43. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$, then $(p + q)$ is equal to 69.

(a) True (b) False
(c) Can't say (d) Partially True/False

44. Column-I denotes the number and Column-II denotes the prime factors of the numbers given in Column-I. Match them correctly.

	Column I		Column II
P.	945	1.	$2 \times 5 \times 11^2 \times 17$
Q.	20570	2.	$7^2 \times 13 \times 17 \times 19$
R.	205751	3.	$3^3 \times 5 \times 7$

	P	Q	R
(a)	3	1	2
(b)	1	3	2
(c)	2	1	3
(d)	1	2	3

45. Match the Column

	Column I		Column II
P.	13/125	1.	Irrational
Q.	$\sqrt{2}$	2.	Terminating decimal expansion
R.	200/25	3.	Non-terminating repeating decimal expansion
S.	61/455	4.	Rational number

	P	Q	R	S
(a)	2	4	1	3
(b)	2	1	4	3
(c)	1	2	3	4
(d)	1	2	4	3

Assertion-Reasoning MCQs

Directions (Q. Nos. 46-54) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true; R is False.
(d) A is false; R is true.

46. **Assertion (A)** HCF of (11, 17) is 1.
Reason (R) If p and q are prime, then HCF $(p, q) = 1$

47. **Assertion (A)** If LCM $= 182$, product of integers is 26×91, then HCF $= 13$.
Reason (R) LCM $\times$ Product of integers = HCF

48. **Assertion (A)** We can say that $3|93$ and $5|0$ is true.
Reason (R) A non-zero integer a is said to divide an integer b if there exists an integer c such that $b = ac$.

49. **Assertion (A)** $n^2 - n$ is divisible by 2 for every positive integer.
Reason (R) $\sqrt{2}$ is not a rational number.

50. **Assertion (A)** $\sqrt{2}$ is an irrational number.
Reason (R) If p be a prime, then $\sqrt{p}$ is an irrational number.

51. **Assertion (A)** 2 is a rational number.
Reason (R) The square roots of all positive integers are irrationals.

52. **Assertion (A)** Denominator of 34.12345. When expressed in the form $\frac{p}{q}$, $q \neq 0$, is of the form $2^m \times 5^n$, where m, n are non-negative integers.
Reason (R) 34.12345 is a terminating decimal fraction.

53. **Assertion (A)** $\frac{13}{3125}$ is a terminating decimal fraction.
Reason (R) If $q = 2^n \cdot 5^m$ where n, m are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.

54. **Assertion (A)** 29/9261 will have a non-terminating repeating decimal expansion.
Reason (R) Let $a = p/q$ be a rational number such that p and q are co-prime and the prime factorization of q is of the form $2^n \times 5^m$, where n and m are non-negative integers (whole numbers). Then, a has a decimal expansion, which is non-terminating repeating.

Case Based MCQs

55. **Divisibility Rules**

HCF and LCM are widely used in number system especially in real numbers in finding relationship between different numbers and their general forms. Also, product of two positive integers is equal to the product of their HCF and LCM.
{Product of numbers = HCF $\times$ LCM}
Based on the above information answer the following questions.

(i) If two positive integers a and b are expressible in terms of primes as $a = p^2q^4$ and $b = p^3q^2$, then which of the following is true?
(a) HCF $= pq^2 \times$ LCM
(b) LCM $= pq^2 \times$ HCF
(c) LCM $= p^2q \times$ HCF
(d) HCF $= p^2q \times$ LCM

(ii) Vishal has a collection of marbles realizes that if he makes a group of 2 or 3 marbles, there are always one marbles left, then which of the following is correct if the number of marbles is p?
(a) p is odd
(b) p is even
(c) can't say
(d) Both (a) and (b)

(iii) Given that HCF (306, 657) = 9, find LCM (306, 657).

(a) 33228 (b) 22833
(c) 22338 (d) None of these

(iv) The greatest number of 6-digits exactly divisible by 15, 24 and 36 is

(a) 999998 (b) 999999
(c) 999720 (d) 999724

(v) If N is the sum of first 13986 prime numbers, then N is always divisible by

(a) 6 (b) 4
(c) 8 (d) None of these

56. To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- Section A and Section B of grade X. There are 32 students in section A and 36 students in section B. ***[CBSE Question Bank]***

Based on the above information answer the following questions.

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

(a) 144 (b) 128 (c) 288 (d) 272

(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true, then the HCF (32, 36) is

(a) 2 (b) 4 c) 6 (d) 8

(iii) 36 can be expressed as a product of its primes as

(a) $2^2 \times 3^2$
(b) $2^1 \times 3^3$
(c) $2^3 \times 3^1$
(d) $2^0 \times 3^0$

(iv) $7 \times 11 \times 13 \times 15 + 15$ is a

(a) Prime number
(b) Composite number
(c) Neither prime nor composite
(d) None of the above

(v) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a and b are prime numbers, then the LCM (p, q) is

(a) ab (b) a^2b^2
(c) a^3b^2 (d) a^3b^3

57. Activity on Real Numbers

In a classroom (class-X) activity on real numbers, the students have to pick a number card from a box and frame question on it is not a rational number for the rest of the class. The number cards picked up by first 5 students and their questions on the numbers for the rest of the class are as shown below.

Based on the above information answer the following questions.

(i) Akshay picked up $\sqrt{12}$ and his question was-Which of the following is true about $\sqrt{12}$

(a) It is a natural number
(b) It is an irrational number
(c) It is a rational number
(d) None of the above

(ii) Shallu picked up 'BONUS' and her question was-Which of the following is not irrational?

(a) $5 - 3\sqrt{2}$
(b) $\sqrt{5} - 2$
(c) $2 + 2\sqrt{9}$
(d) $3\sqrt{5} - 6$

(iii) Charu picked up $\sqrt{3}-\sqrt{2}$ and her question was $\sqrt{3}-\sqrt{2}$ is ______ number.

(a) a natural (b) an irrational
(c) a whole (d) a rational

(iv) Bhoomika picked up $\frac{1}{\sqrt{8}}$ and her question was $\frac{1}{\sqrt{8}}$ is ______ number.

(a) a whole (b) a rational
(c) an irrational (d) a natural

(v) Malika picked up $\sqrt{5}$ and her question was - Which of the following is not irrational?

(a) $15+3\sqrt{5}$ (b) $\sqrt{20}-9$
(c) $4\sqrt{129}$ (d) $\sqrt{16}$

58. A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. ***[CBSE Question Bank]***

Based on the above information answer the following questions.

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number participants that can accommodated in each room are

(a) 14 (b) 12
(c) 16 (d) 18

(ii) What is the minimum number of rooms required during the event?

(a) 11 (b) 31
(c) 41 (d) 21

(iii) The LCM of 60, 84 and 108 is

(a) 3780 (b) 3680
(c) 4780 (d) 4680

(iv) The product of HCF and LCM of 60, 84 and 108 is

(a) 55360 (b) 35360
(c) 45500 (d) 45360

(v) 108 can be expressed as a product of its primes as

(a) $2^3\times3^2$ (b) $2^3\times3^3$
(c) $2^2\times3^2$ (d) $2^2\times3^3$

59. Decimal Expansion

Decimal form of rational numbers can be classified into two types.

- Let a be a rational number whose decimal expansion terminates. Then a can be expressed in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n\cdot5^m$, where n, m are non-negative integers and vice-versa.
- Let $a=\frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n and m are non-negative integers. Then a has a non-terminating repeating decimal expansion.

Based on the above information answer the following questions.

(i) Which of the following rational numbers have a terminating decimal expansion?

(a) 8/15 (b) 51/150
(c) 15/400 (d) $129/(2^2\times5^2\times7^2)$

(ii) $23/(2^3 \times 5^3) =$

(a) 0.575 (b) 0.023
(c) 0.91 (d) 1.15

(iii) $686/(2^2 \times 5^7 \times 7^3)$ is a ________ decimal.

(a) terminating
(b) recurring
(c) non-terminating and non-recurring
(d) None of the above

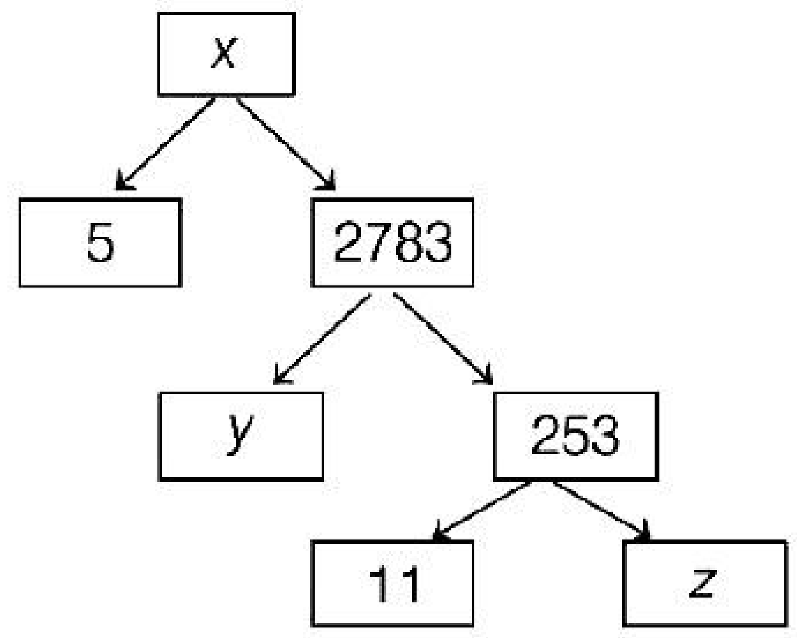
(iv) For which of the following value(s) of p, $251/(2^3 \times p^2)$ is terminating decimal number?

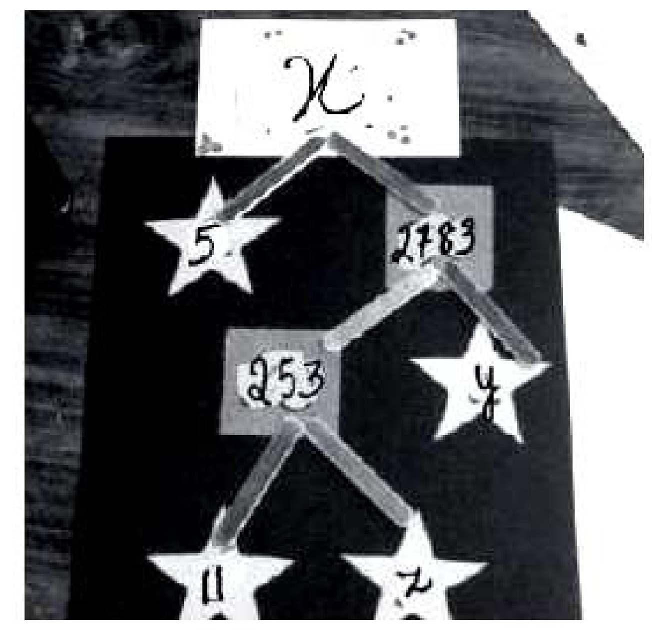
(a) 3 (b) 7
(c) 5 (d) All of these

(v) $61/(2^5 \times 5^3)$ is a _________ decimal

(a) terminating
(b) recurring
(c) non-terminating and non-recurring
(d) None of the above

60. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and ask for your help in completing a quiz for the audience. Observe the following factor tree and answer the following :





[CBSE Quesion Bank]

Based on the above information answer the following questions.

(i) What will be the value of x ?

(a) 15005 (b) 13915
(c) 56920 (d) 17429

(ii) What will be the value of y ?

(a) 23 (b) 22 (c) 11 (d) 19

(iii) What will be the value of z ?

(a) 22 (b) 23 (c) 17 (d) 19

(iv) According to Fundamental Theorem of Arithmetic 13915 is a

(a) Composite number
(b) Prime number
(c) Neither prime nor composite
(d) Even number

(v) The prime factorisation of 13915 is

(a) $5 \times 11^3 \times 13^2$ (b) $5 \times 11^3 \times 23^2$
(c) $5 \times 11^2 \times 23$ (d) $5 \times 11^2 \times 13^2$

ANSWERS

Multiple Choice Questions

1. (c)	*2. (d)*	*3. (a)*	*4. (b)*	*5. (c)*	*6. (b)*	*7. (b)*	*8. (b)*	*9. (a)*	*10. (c)*
11. (a)	*12. (b)*	*13. (b)*	*14. (b)*	*15. (a)*	*16. (b)*	*17. (d)*	*18. (b)*	*19. (b)*	*20. (c)*
21. (a)	*22. (a)*	*23. (b)*	*24. (b)*	*25. (c)*	*26. (c)*	*27. (a)*	*28. (c)*	*29. (c)*	*30. (c)*
31. (d)	*32. (a)*	*33. (a)*	*34. (b)*	*35. (d)*	*36. (a)*	*37. (d)*	*38. (b)*	*39. (c)*	*40. (c)*
41. (d)	*42. (a)*	*43. (a)*	*44. (a)*	*45. (b)*					

Assertion-Reasoning MCQs

46. (a)	*47. (c)*	*48. (a)*	*49. (b)*	*50. (a)*	*51. (c)*	*52. (a)*	*53. (a)*	*54. (c)*

Case Based MCQs

55. (i) (b) (ii) (a) (iii) (c) (iv) (c) (v) (d)
56. (i) (c) (ii) (b) (iii) (a) (iv) (b) (v) (b)
57. (i) (b) (ii) (c) (iii) (b) (iv) (c) (v) (d)
58. (i) (b) (ii) (d) (iii) (a) (iv) (d) (v) (d)
59. (i) (c) (ii) (b) (iii) (a) (iv) (c) (v) (a)
60. (i) (b) (ii) (c) (iii) (b) (iv) (a) (v) (c)

SOLUTIONS

1. We know that, even integers are 2, 4, 6, ...
So, it can be written in the form of $2m$, where, m = Integer = z

[since, integer is represented by z]

or $\quad m = \ldots, -1, 0, 1, 2, 3, \ldots$

$\therefore \quad 2m = \ldots, -2, 0, 2, 4, 6, \ldots$

2. Since, n is divisible by 2, therefore $(n+2)$ is divisible by 4, and hence $n(n+2)$ is divisible by 8.
Also, $n, n+1, n+2$ are three consecutive numbers.
So, one of them is divisible by 3.
Hence, $n(n+1)(n+2)$ must be divisible by 24.

3. Yes, two consecutive integers can be $n, (n+1)$. So, one number of these two must be divisible by 2. Hence, product of numbers is divisible by 2.

4. Given, n is a natural number. Let us assume that 4^n ends with 0, thus 4^n is divisible by 2 and 5 both.
But prime factors of 4 are 2×2.

$\therefore \quad 4^n = (2 \times 2)^n = 2^{2n}$

Thus, prime factorisation of 4^n does not contain 5. So, the fundamental theorem of arithmetic guarantees that there are no other primes in the factorisation of 4^n.
So, our assumption that 4^n ends with 0 is wrong.

5. If any number ends with the digit 0 or 5, it is always divisible by 5.
If 12^n ends with the digit zero it must be divisible by 5.
This is possible only if prime factorisation of 12^n contains the prime number 5.
Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow \quad 12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

[since, there is no term contains 5]

Hence, there is no value of $n \in N$ for which 12^n ends with digit zero or five.

6. We have, $(3 \times 5 \times 7) + 7 = 105 + 7 = 112$

$\therefore$ Prime factors of $112 = 2 \times 2 \times 2 \times 2 \times 7$
$= 2^4 \times 7$

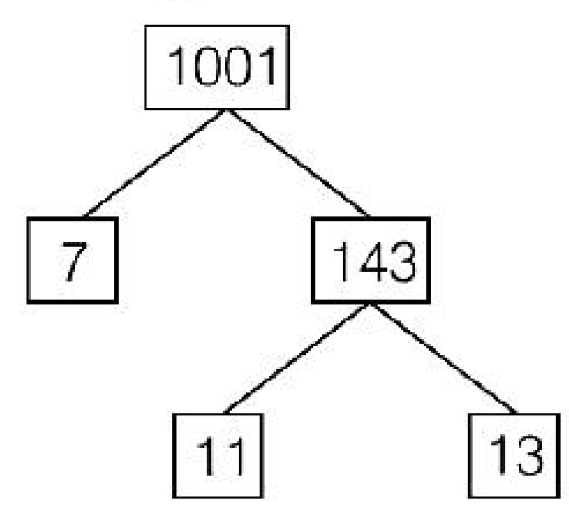
So, it is the product of prime factors 2 and 7. i.e. it has factors other than 1 and itself. Hence, it is a composite number.

7. $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 5$
$= 5040 + 5 = 5045 = 5 \times 1009$

Thus, it is the product of prime factor of 5 and 1009.
Hence, it is a composite number.

8. A prime number has exactly two factors, 1 and itself
So, total number of factors of a prime number is 2.

9. Given number is 10001. Then, the factor tree of 1001 is given as below



$\therefore \quad 1001 = 7 \times 11 \times 13$

By comparing with given factor tree, we get
$x = 7, y = 13$

10. We have,
$96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

Hence, HCF $(96, 404) = 2^2 = 4$

11.

2	27300
2	13650
3	6825
5	2275
5	455
7	91
	13

$\therefore$ $27300 = 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 13$

Total number of distinct primes are 5 which are 2, 3, 5, 7 and 13.

12. HCF = Product of lowest powers of common factors

$= 3 \times 5 \times 7$

$= 105$

13. Given that, $a = x^3 y^2 = x \times x \times x \times y \times y$

and $b = xy^3 = x \times y \times y \times y$

$\therefore$ HCF of a and b = HCF (x^3y^2, xy^3)

$= x \times y \times y = xy^2$

[since, HCF is the product of the smallest power of each common prime facter involved in the numbers]

14. Smallest composite number $= 4 = 2 \times 2 = 2^2$

and smallest prime number $= 2 = 2^1$

$\therefore$ HCF $(4, 2) = 2^1 = 2$

[since, HCF is the product of the smallest power of each common prime factor involved in the numbers]

15. Let the required numbers be $15x$ and $11x$.

Then, their HCF is x.

So, $x = 13$

$\therefore$ The numbers are 15×13 and 11×13

i.e. 195 and 143.

16. Product of two coprime numbers is equal to their LCM.

So, LCM = 117

17. Factors of 1 to 10 numbers

$1 = 1$

$2 = 1 \times 2$

$3 = 1 \times 3$

$4 = 1 \times 2 \times 2$

$5 = 1 \times 5$

$6 = 1 \times 2 \times 3$

$7 = 1 \times 7$

$8 = 1 \times 2 \times 2 \times 2$

$9 = 1 \times 3 \times 3$

$10 = 1 \times 2 \times 5$

$\therefore$ LCM of numbers 1 to 10

$=$ LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

18. By prime factorisation method, factors of 65 and 117 are given as,

$65 = 5 \times 13$

and $117 = 3^2 \times 13$

$\therefore$ HCF $(65, 117) = 13$ ...(i)

According to the question, we have

$65m - 117 = \text{HCF } (65, 117)$

$\Rightarrow$ $65m - 117 = 13$

$65m = 13 + 117$

$65m = 130$

$\Rightarrow$ $m = 2$

19. Factors of 12 and 42 are

$12 = 2 \times 2 \times 3$

$42 = 2 \times 3 \times 7$

Hence, LCM $(12, 42) = 2 \times 2 \times 3 \times 7 = 84$

And $84 = 10 \times 8 + 4$

Therefore, $m = 8$

20. Let x be number of fruits in one row such that each fruit contains same number of fruits.

i.e. x = HCF of (24, 36, 60)

The factors of 24, 36 and 36 are

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

$x = 2 \times 2 \times 3 = 12$

So, peaches have $12 \times 2 = 2$ rows, apricots have $12 \times 3 = 3$ rows and bananas have $12 \times 5 = 60$, 5 rows.

Minimum number of rows $= 2 + 3 + 5 = 10$

21. The time interval between simultaneous tolling of the bells = LCM (10, 15, 20, 30) s

$= 60$ s $= 1$ min

Hence, the bells will toll together again for the first time after 10 : 00 am at 10 : 01 am.

22. P is the LCM of 2, 4, 6, 8, 10

$\therefore$ $P = 3(8)(5)$

Q is the LCM of 1, 3, 5, 7, 9

$\therefore$ $Q = 5(7)(9)$

L is the LCM of P, Q

$\therefore\ L = 3(8)(5)\,21$ or $5(7)(9)\,8$

i.e. $L = 21P$ or $8Q$

23. The prime factorisation of

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 2^3 \times 3 \times 5$$

and prime factorisation of 144

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^4 \times 3^2$$

Now, LCM $(120, 144) = 2^4 \times 3^2 \times 5 = 720$

and HCF $(120, 144) = 2^3 \times 3 = 24$

24. Let the two number are a and b.

Given, HCF $(a, b) = 2$

and product $(a \times b) = 120$

We know that,

HCF $(a, b) \times$ LCM (a, b) = Product of a and b

$\therefore\ 2 \times \text{LCM}\,(a, b) = 120$

$\Rightarrow\ \text{LCM}\,(a, b) = \dfrac{120}{2} = 60$

Hence, the required LCM is 60.

25. We know that,

HCF $\times$ LCM = Product of two numbers

$\Rightarrow\ 45 \times a \times 13 = 65 \times 117$

$\Rightarrow\ a = \dfrac{65 \times 117}{45 \times 13} \Rightarrow a = 13$

26. We know that

HCF $\times$ LCM = Product of two numbers

$\Rightarrow 12 \times \text{LCM}\,(a, b) = 1800$

$\Rightarrow\ \text{LCM}\,(a, b) = \dfrac{1800}{12} = 150$

27. We know that,

LCM $\times$ HCF = Product of two numbers

$$\text{LCM} \times 9 = 306 \times 675$$
$$\text{LCM} = \frac{306 \times 675}{9}$$
$$= 34 \times 657 = 22338$$

$\therefore$ LCM of 306 and 657 = 22338

28. Given, LCM = 1001 and HCF = 7

Let the two numbers be x and y.

$\therefore\ x = 7c$ and $y = 7b$, where c and b are coprimes.

We have, $x \times y$ = LCM $\times$ HCF

$\Rightarrow\ 7a \times 7b = 1001 \times 7$

$\Rightarrow\ ab = 143$

$\Rightarrow\ (a, b) = (1, 143)$ or $(11, 13)$

There are two pairs of numbers.

29. Since, p is prime.

$\therefore\ p$ and $p + 1$ has no common factor other than 1.

$\therefore$ HCF of p and $p + 1 = 1$

and LCM of p and $p + 1 = p \times (p + 1)$

$= p(p + 1)$

30. We have, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\because$ HCF = product of smallest power of each common prime factor in the number = 3

and LCM = product of greatest power of each prime factor involved in the number $= 2^2 \times 3 \times 5 \times 7 = 420$

Thus, HCF and LCM are 3 and 420, respectively.

31. (a) $\dfrac{22}{7} = 3.14285714285\ldots = 3.\overline{14285}$

(b) 3.1416 is not irrational number, because it is terminating decimal number.

(c) $3.\overline{1416}$ is not irrational number, because it is non-terminating repeating decimal number.

which is not irrational number.

(d) 3.141441444 ... is an irrational number, because it is non-terminating repeating decimal number.

32. There is infinite numbers between any two numbers.

So, there must be infinite irrational numbers too.

33. Product of a non-zero rational and an irrational number is always irrational i.e.
$\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ (irrational).

34. The number is non-terminating recurring.
So, it is a rational number.

35. p/q has a terminating decimal expansion, if prime factorization of q has 2 or 5 or both.

36. Given rational number is $\frac{13}{125}$.
$\therefore$ Prime factorisation of $125 = 5^3$
Now, $\frac{13}{125} = \frac{13}{5^3} = \frac{13 \times 2^3}{5^3 \times 2^3} = \frac{104}{10^3} = 0.104$
[make the denominator in the power of 10]
Thus, $\frac{13}{125}$ has decimal expansion, which terminates.

37. We have, $\frac{1458}{1250} = \frac{1458}{2 \times 5^4}$
$$= \frac{1458 \times 2^3}{2 \times 5^4 \times 2^3} = \frac{11664}{(2 \times 5)^4}$$
$$= \frac{11664}{10^4} = \frac{11664}{10000}$$
$$= 1.1664$$
So, it will terminates after 4 places of decimals.

38. $\frac{125}{2^4 \cdot 5^3} = \frac{125 \times 5}{(2 \cdot 5)^4} = \frac{625}{10000}$

39. We have, $\frac{51}{1500} = \frac{17}{500}$
Prime factorization of 500

2	500
2	250
5	125
5	25
5	5
	1

$$= 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$$
which is in the form $2^n \times 5^m$.
So, it has a terminating decimal expansion.
Now, $\frac{51}{1500} = \frac{17}{2^2 \times 5^3}$
By comparing, we get $n = 2$ and $m = 3$
$\therefore \quad m + n = 2 + 3 = 5$

40. (a) Here, denominator $= 210 = 2 \times 3 \times 5 \times 7$, which is not the form of $2^n 5^m$, so it is not terminating.
(b) Here, denominator $= 2^2 \times 5^7 \times 7^5$, which is not the form of $2^n\ 5^m$, so it is not terminating.
(c) Here, denominator $= 3125 = 5^5$ which is the form of $2^n\ 5^m$, so it is terminating.
Hence, option (c) is correct.

41. 3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (m and n are whole numbers), so all fractions have terminating decimal expansion.

42. Here, denominator $= 164 = 2 \times 2 \times 41$, which is not the form of $2^n \times 5^m$, so it is non-terminating.

43. Let $x = 0.2\overline{54}$, then
$x = 0.2545454\ldots$ ...(i)
On multiplying Eq. (i) by 100, we get
$\therefore \quad 100x = 25.4545\ldots$ ...(ii)
On subtracting Eq. (i) from Eq. (ii), we get
$$99x = 25.2$$
$\Rightarrow \quad x = \frac{252}{990} = \frac{14}{55}$
On comparing with $\frac{p}{q}$, we get
$$p = 14 \text{ and } q = 55$$
$\therefore \quad p + q = 14 + 55 = 69$

44.

3	945
3	315
3	105
5	35
7	7
	1

Prime factors of $945 = 3^3 \times 5 \times 7$

2	20570
5	10285
11	2057
11	187
17	17
	1

Prime factors of $20570 = 2 \times 5 \times 11^2 \times 17$

7	205751
7	29393
13	4199
17	323
19	19
	1

Prime factors of $205751 = 7^2 \times 13 \times 17 \times 19$

45. (P) $\frac{13}{125} = \frac{13}{5^3}$

$125 = 5^3$ is of the form $2^n \times 5^m$

So, it is terminating decimal expansion.

(Q) $\sqrt{2}$ is irrational

(R) $\frac{200}{25} = 8$, it can be represented as p/q form, so it is a rational number.

(S) $\frac{61}{455} = \frac{61}{5 \times 7 \times 13}$

$455 = 5^1 \times 13 \times 7$ is not of the form $2^n \times 5^m$.

So, it is non-terminating repeating decimal expansion.

46. As both the given numbers $(11, 17)$ are prime numbers

$\therefore$ HCF of $(11, 17) = 1$

Assertion is true, Reason is true and Reason is the correct explanation of Assertion.

47. LCM $\times$ HCF = Product of numbers

$\Rightarrow$ $182 \times \text{HCF} = 26 \times 91$

$\Rightarrow$ $\text{HCF} = \frac{26 \times 91}{182} = 13$

$\therefore$ Assertion is true but Reason is false.

48. If $3|93$ means 3 divides 93

$\Rightarrow$ $93 = a \times 3$ (a is an integer)

Similarly, if $5|0$ means 5 divides 0

$\Rightarrow$ $0 = 5 \times b$ (b is an integer)

$\therefore$ Assertion is true Reason is true and Reason is the correct explanantion of Assertion.

49. **Case I** If $n = 2q$

$$n^2 - n = 4q^2 - 2q$$
$$= 2q(2q-1) \quad \{\text{divisible by } 2\}$$

Case II If $n = 2q + 1$

$$n^2 - n = (2q+1)^2 - (2q+1)$$
$$= 4a^2 + 4q + 1 - 2q - 1$$
$$= 2q(2q+1) \quad \{\text{divisible by } 2\}$$

Hence, Assertion and Reason both are true but Reason is not a correct explanation of Assertion.

50. Assertion and Reason both are true and Reason is the correct explanation of Assertion.

51. $\because \sqrt{4} = \pm 2$, which is not an irrational number.

$\therefore$ Assertion is true but Reason is false.

52. Reason is clearly true.

Again $34.12345 = \frac{3412345}{100000} = \frac{682469}{20000}$

$= \frac{682469}{2^5 \times 5^4}$

Its denominator is of the form $2^m \times 5^n$

$\left[\begin{array}{c} m = 5, n = 4 \text{ are} \\ \text{non-negative integers} \end{array}\right]$

Assertion and Reason both are true and Reason is the correct explanation of Assertion.

53. Since, the factors of the denominator 3125 is of the form $2^0 \times 5^5$.

$\therefore \frac{13}{3125}$ is a terminating decimal.

Assertion and Reason both are true and Reason is the correct explanation of Assertion.

54. Given number $= \dfrac{29}{9261} = \dfrac{29}{3^3 \times 7^3}$

$9261 = 3^3 \times 7^3$ is not of the form $2^n \times 5^m$.

Hence, $\dfrac{29}{9261}$, is non-terminating repeating decimal expansion.

Assertion is true but Reason is false.

55. (i) The factors are

$$a = p^2q^4$$
$$b = p^3q^2$$

HCF of a and $b = p^2q^2$

LCM of a and $b = p^3q^4$

$\therefore$ LCM $= pq^2 \times$ HCF

(ii) Let the number of marbles be $2m+1$ or $3n+1$

$\therefore$ Total number of marbles be in the form of (multiple of 2 and 3) + 1

or $p = 6a + 1$ {a is any integer}

p is an odd number.

(iii) Here, HCF of 306 and 657 = 9, we have

LCM × HCF = Product of the numbers

$\therefore$ LCM $\times 9 = 306 \times 657$

$\Rightarrow$ LCM $= \dfrac{306 \times 657}{9} = 34 \times 657$
$= 22338$

Thus, LCM of 306 and 657 is 22338.

(iv) LCM of 15, 24 and 36 is 360.

Greatest number of 6-digit is 999999.

So, $999999 = 360 \times 2777 + 279$

$\therefore$ Greatest number of 6-digits divisible by 15, 24 and 36 is $999999 - 279 = 999720$.

(v) N will be odd number because N is sum of one even number and 13986 odd numbers.

Hence, N will not be divisible by any even number.

56. (i) Given, number of students in Section A $= 32$

Number of students in Section B = 36

The minimum number of books acquire for the class library = LCM of (32, 36)

$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^5 \times 3^2 = 32 \times 9 = 288$

(ii) Given, product of the two numbers
= LCM × HCF

$\therefore$ $32 \times 36 =$ LCM (32, 36) × HCF (32, 36)

$\Rightarrow$ $32 \times 36 = 288 \times$ HCF (32, 36)

$\Rightarrow$ HCF (32, 36) $= \dfrac{32 \times 36}{288} = 4$

(iii) The prime factors of 36 are

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

(iv) $7 \times 11 \times 13 \times 15 + 15 = 15015 + 15$
$= 15030$

Hence, it is a composite number.

(v) Given, $p = ab^2$ and $q = a^2b$

LCM (p, q) = Product of the greatest power of each prime factor involved in the numbers, with highest power

$= a^2 \times b^2$

57. (i) Given, $\sqrt{12}$ (a number)

$$\sqrt{12} = 2\sqrt{3}$$

Here, 2 is a rational number and $\sqrt{3}$ is an irrational number.

We know that, product of rational and irrational number is a irrational number.

Hence, $\sqrt{12}$ is an irrational number.

(ii) Here, $\sqrt{9} = 3$ (which is a rational number)

$\therefore 2 + 2\sqrt{9} = 2 + 2(3)$
$= 2 + 6 = 8$

8 is not an irrational number.

(iii) As we know, $\sqrt{2}$ and $\sqrt{3}$ both are irrational number and the difference of two irrational number is always a irrational number.

So, $\sqrt{3} - \sqrt{2}$ is an irrational number.

(iv) $\because \sqrt{8} = 2\sqrt{2}$

$2\sqrt{2}$ is an irrational number and the reciprocal of irrational number is also a irrational number.

So, $\frac{1}{\sqrt{8}}$ is an irrational number.

(v) Here, $\sqrt{16} = 4$

4 is a rational number or not irrational.

58. (i) Given, number of students in each subject are

Hindi $= 60$, English $= 84$,

Mathematics $= 108$.

The prime factors of each subject students are

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$
$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$
$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$

The maximum number of participants that can accommodated in each room

$= \text{HCF } (60, 84, 108)$

= Product of the smallest power of each common prime factor involved in the numbers

$= 2^2 \times 3 = 12$

(ii) The minimum number of rooms required during the event is

$$\frac{\text{Total number of participants}}{12} = \frac{252}{12} = 21$$

(iii) LCM of $(60, 84, 108)$ = Product of the greatest power of each prime factor involved in the numbers with highest power

$$= 2^2 \times 3^3 \times 5 \times 7$$
$$= 4 \times 27 \times 35$$
$$= 3780$$

(iv) Now, HCF $(60, 84, 108)$

$\times$ LCM $(60, 84, 108)$

$= 12 \times 3780$

$= 45360$

(v) Number 108 can be expressed as a product of its prime as $2^2 \times 3^3$.

59. (i) The simplest form of are the options can be written as

$$\frac{8}{15} = \frac{2^3}{3 \times 5}$$
$$\frac{51}{150} = \frac{51}{2 \times 3 \times 5^2}$$
$$\frac{15}{400} = \frac{3 \times 5}{2^4 \times 5^2}$$
$$= \frac{129}{2^2 \times 5^2 \times 7^2}$$

Only (c) has denominator in the form of $2^n 5^m$.

(ii) $\frac{23}{2^3 \times 5^3} = \frac{23}{10^3} = \frac{23}{1000} = 0.023$

(iii) $\frac{686}{2^2 \times 5^7 \times 7^3} = \frac{2 \times 7^3}{2^2 \times 5^7 \times 7^3} = \frac{2}{2^2 \times 5^7}$

denominator is in the form of $2^n 5^m$ so terminating.

(iv) For the fraction to be terminating the denominator must be in the form of $2^n 5^m$, so the values of P must be 2 or 5.

Hence, (c) will be correct.

(v) $\frac{61}{2^5 \times 5^3}$ here the denominator have only 2 or 5 prime factor (or in the form of $2^n 5^m$), so $\frac{61}{2^5 \times 5^3}$ is a terminating decimal.

60. (i) $x = 5 \times 2783 = 13915$

(ii) We have, $2783 = y \times 253$

$$\Rightarrow \quad y = \frac{2783}{253} = 11$$

(iii) We have, $253 = 11 \times z$

$$\Rightarrow \quad z = \frac{253}{11} = 23$$

(iv) According to the fundamental theorem of Arithmetic 13915 is a composite number.

(v) $13915 = 5 \times 11 \times 11 \times 23$

$= 5 \times 11^2 \times 23$